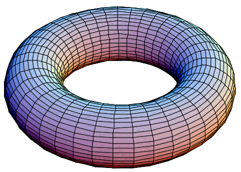
Adaptive Control of Feedback Linearizable Systems

Harry G. Kwatny

Department of Mechanical Engineering & Mechanics

Drexel University



Outline

- Setup
- Ideal vs. actual control
- Affine systems
- Non-affine systems
- Example

Setup

- Parameter-dependent system reduce to normal form

$$\dot{\xi} = F(\xi, z, \vartheta)$$

$$\dot{z} = Az + E\left[\alpha(z,\vartheta) + \rho(z,\vartheta)u\right]$$

$$y = Cz$$

- Design a feedback control that guarantees output stabilization $y \to 0$ for all ϑ

Ideal vs. Actual Control

- Ideal control is based on actual parameter

$$u^* = \rho^{-1}(z,\vartheta)\left\{-\alpha(z,\vartheta)+v\right\}$$

- Actual control is based on parameter estimate

$$u = \rho^{-1}\left(z,\hat{\vartheta}\right)\left\{-\alpha\left(z,\hat{\vartheta}\right)+v\right\}$$

System Dynamics with Actual Control

$$\dot{z} = Az + E\left[\alpha\left(z,\vartheta\right) + \rho\left(z,\vartheta\right)u\right]$$

$$\Downarrow$$

Ideal control $\qquad \dot{z} = Az + E\left[v + \Delta\right] \qquad$ Actual control

$$\Delta = \left[\alpha\left(z,\vartheta\right) + \rho\left(z,\vartheta\right)u\right] - \left[\alpha\left(z,\hat{\vartheta}\right) + \rho\left(z,\hat{\vartheta}\right)u\right]$$

Assumption 1: $\Delta\left(\xi, z, \hat{\vartheta}, \vartheta, u\right) = \Psi\left(\xi, z, \hat{\vartheta}, u\right)\left(\vartheta - \hat{\vartheta}\right)$

$$\Downarrow$$

$$\dot{z} = A_c z + E\Psi\left(\vartheta - \hat{\vartheta}\right)$$

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Proposition 1

- Asymptotic output stabilization is achieved with the parameter estimator

$$\dot{\hat{\vartheta}} = Q\Psi^T\left(\xi, z, \hat{\vartheta}, u\right)E^T Pz, \quad Q = Q^T > 0$$

- Where *P* is the symmetric, positive definite solution of

$$A_c^T P + PA_c = -I$$

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Proof of Prop 1

- Choose candidate Lyapunov function

$$V = z^T P z + \left(\vartheta - \hat{\vartheta}\right)^T Q^{-1} \left(\vartheta - \hat{\vartheta}\right)$$

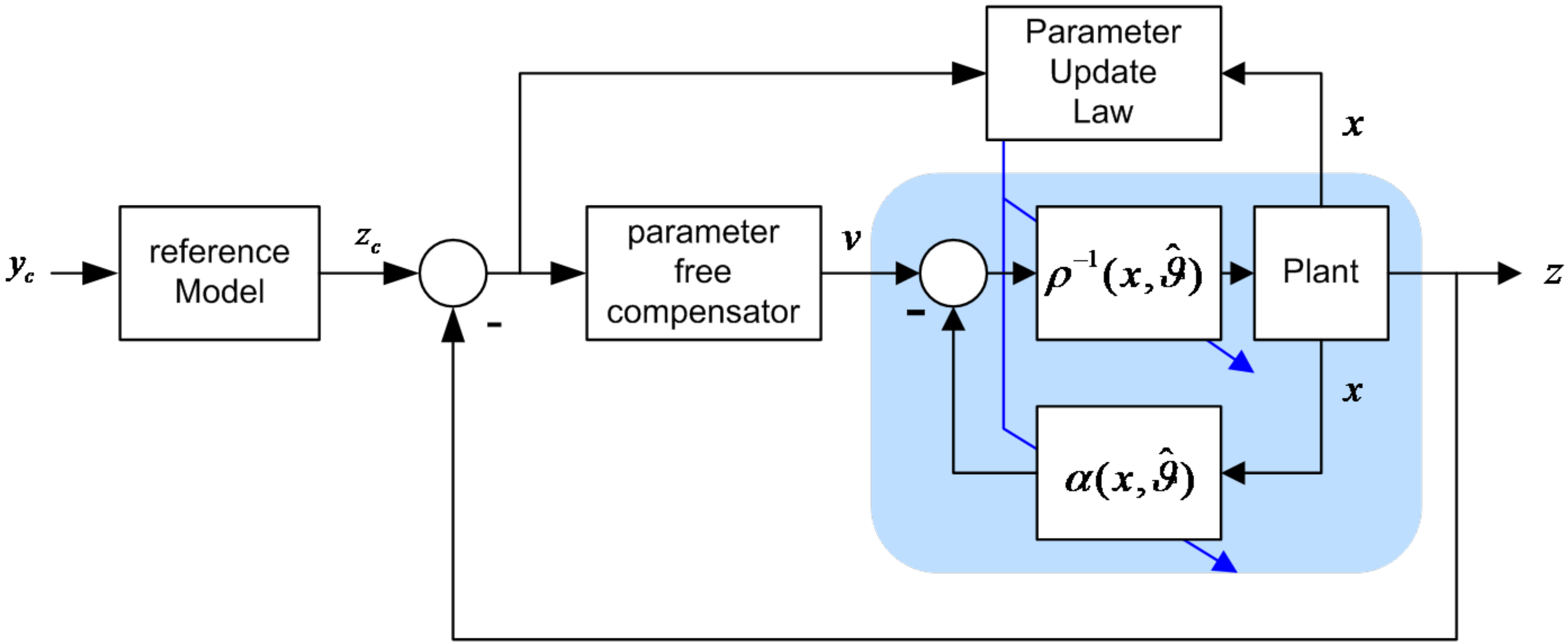
- Compute

$$\dot{V} = 2z^T P\dot{z} - 2\dot{\hat{\vartheta}}^T Q^{-1}\left(\vartheta - \hat{\vartheta}\right)$$

$$= z^T \left(PA_c + A_c^T P\right) z + 2\left(z^T PE\Psi - \dot{\hat{\vartheta}}^T Q^{-1}\right)$$

$$= -z^T z$$

Adaptive Control Structure



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Example

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}x_2\\ -0.2x_2+x_1^3/2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}\left[u+\kappa x_1^3+au\right]$$

$$\kappa\in[0,1],\quad a\in[-0.1,0.1]$$

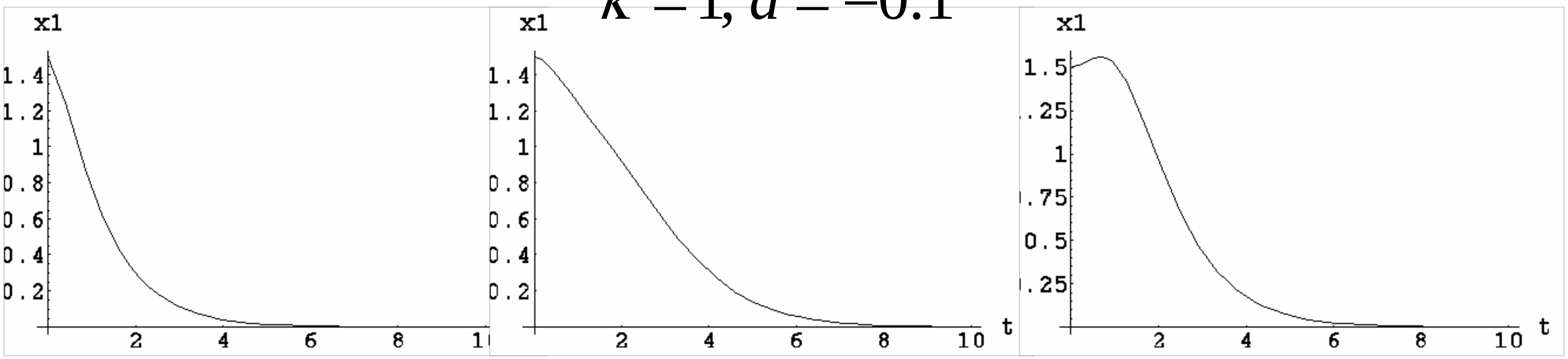
$$\alpha=-0.1x_2+x_1+x_1^3/2,\ \rho=1$$

$$u^*=\rho^{-1}\left(-\alpha+\begin{bmatrix}-1 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}\right)=-2x_1-1.9x_2-x_1^3/2$$

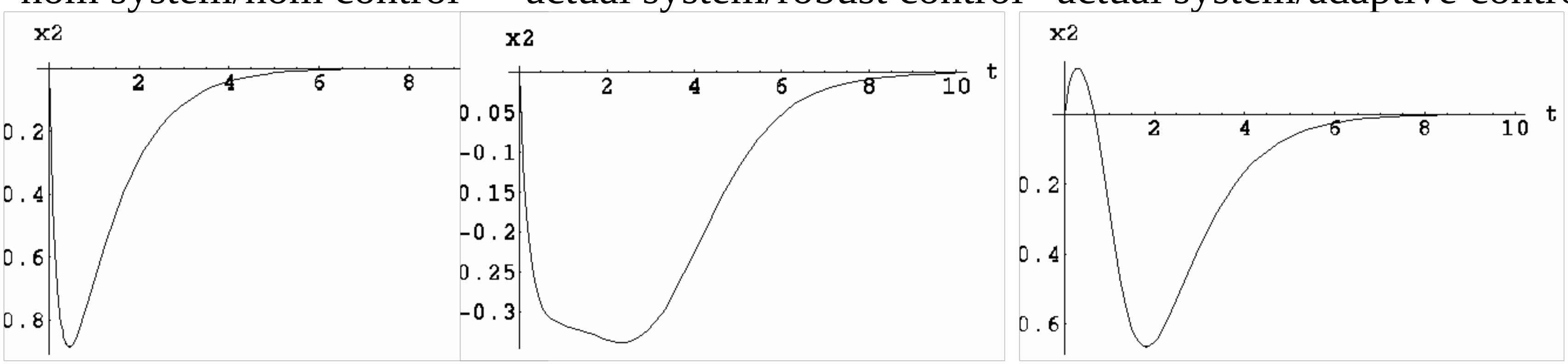
$$\Delta=(\kappa-\hat{\kappa})x_1^3+(a-\hat{a})\left(-2x_1-\frac{x_1^3}{2}-1.9x_1\right)$$

Example ~ Results

$$\kappa = 1, a = -0.1$$



nom system/nom control   actual system/robust control   actual system/adaptive control



Non-affine Systems

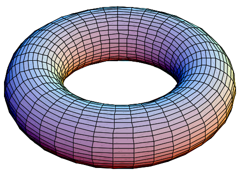
- Consider a system of the form

Non-affine part

$$\dot{\xi} = F\left(\xi, z, \vartheta\right)$$

$$\dot{z} = Az + E\left[\alpha\left(z, \vartheta\right) + \rho\left(z, \vartheta\right)\phi\left(u, \vartheta\right)\right]$$

$$y = Cz$$



- The map ϕ is piecewise smooth and ha
piecewise continuous inverse in *u*.

Ideal and Actual Controls

- The ideal control is

$$u = \varphi^{-1}\left(\rho^{-1}(x, \vartheta)\left(-\alpha(x, \vartheta) + v\right), \vartheta\right)$$

- The actual control is

$$\hat{u} = \phi^{-1}(\rho^{-1}(\hat{x}, \hat{\vartheta})(-\alpha(\hat{x}, \hat{\vartheta}) + Kz), \hat{\vartheta})$$

Actual System Dynamics

- Again

$$\dot{z} = Az + E\left[v + \Delta\right]$$

- With

$$\Delta = [\alpha(x,\vartheta) + \rho(x,\vartheta)\phi(u,\vartheta)] - [\alpha(\hat{x},\hat{\vartheta}) + \rho(\hat{x},\hat{\vartheta})\phi(u,\hat{\vartheta})]$$

- Assumption

$$\Delta\left(\xi, z, \hat{\vartheta}, \vartheta, u\right) = \Psi\left(\xi, z, \hat{\vartheta}, u\right)\left(\vartheta - \hat{\vartheta}\right) + \varphi_0\left(\xi, z, \hat{\vartheta}, \vartheta, u\right)$$

$$\varphi_0 \; bounded, \quad \vartheta_{i,\min} \le \vartheta_i \le \vartheta_{i,\max} \;\; i = 1, \ldots, p$$

Adaptive Control

- Parameter update rule

$$\dot{\hat{\vartheta}} = \Omega \Psi^{\mathrm{T}}\left(\xi, z, \hat{\vartheta}, u\right) E^{\mathrm{T}} P z - \Omega \sigma\left(\hat{\vartheta}\right)$$

- Where

$$(A + EK)^{\mathrm{T}} P + P(A + EK) = -Q, \quad Q > 0$$

- and

$$\sigma(\hat{\vartheta}) = \begin{bmatrix} \sigma_1(\hat{\vartheta}_1) \\ \vdots \\ \sigma_p(\hat{\vartheta}_p) \end{bmatrix}, \sigma_i\left(\hat{\vartheta}_i\right) := \begin{cases} \kappa_i & \hat{\vartheta}_i > \vartheta_{i_{\max}} \\ 0 & \vartheta_{i_{\min}} \leq \hat{\vartheta}_i \leq \vartheta_{i_{\max}} \\ -\kappa_i & \hat{\vartheta}_i < \vartheta_{i_{\min}} \end{cases} \quad \kappa_i > 0$$

Proposition (output convergence)

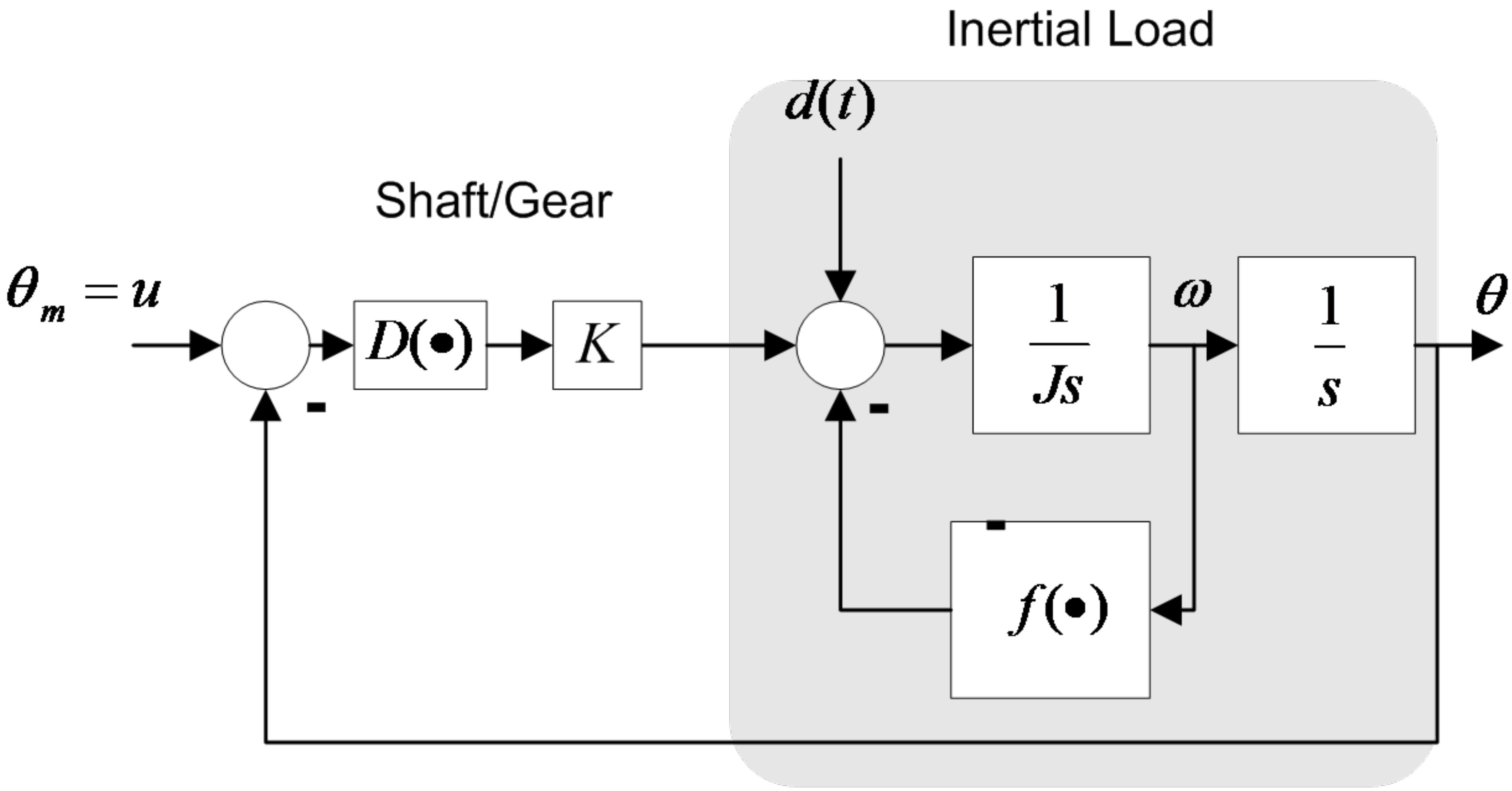
- Define sets $S_{ext} = \left\{ (z, \vartheta) \in R^{r+p} \,\middle|\, \left\| Q^{1/2} z - Q^{-1/2} PE\varphi_0 \right\|^2 > \left\| Q^{-1/2} PE\varphi_0 \right\|^2 \right\}$

$$S_{int} = \left\{ (z, \vartheta) \in R^{r+p} \,\middle|\, \left\| Q^{1/2} z - Q^{-1/2} PE\varphi_0 \right\|^2 < \left\| Q^{-1/2} PE\varphi_0 \right\|^2 \right\}$$

- Basic assumptions and (i) the only invariant set of closed loop on common set boundary corresponds to z=0, (ii) all trajectories that start in C_{param} remain in C_{param}.

$$\Rightarrow z(t) \to 0 \; as \; t \to \infty$$

Example: drive with deadzone & friction



Model

$$T = K\left(\theta_m - \theta\right) \Rightarrow \theta_m = \frac{T}{K} + \theta$$

$$\dot{\theta} = \omega$$

$$\dot{\omega} = -f\left(\omega, b\right) + D\left(T, \varepsilon\right) + \kappa w(t)$$

$$f\left(\omega, b\right) = \begin{cases} b & \omega > 0 \\ -b & \omega < 0 \end{cases} = bsign\left(\omega\right)$$

$$D\left(T, \varepsilon\right) = \begin{cases} x - \varepsilon & x \geq \varepsilon \\ x + \varepsilon & x \leq -\varepsilon \end{cases}$$

Controller ~ 1

- Dead zone inverse

$$D^{-1}(x,\varepsilon)=\begin{cases}x+\varepsilon & x>0\\ x-\varepsilon & x<0\end{cases}$$

- FBL

$$T=D^{-1}\left(-\sqrt{2}\,2\pi\omega-2\pi\theta+f(\omega,b)-\kappa w(t),\varepsilon\right)$$

- Regressor

$$\Psi=-\left[sign(\omega)\quad sign(T)\quad -w(t)\right]$$

$$\Delta=\left[f(\omega,b)+D(T,\varepsilon)+\kappa w(t)\right]-\left[f(\omega,\hat{b})+D(\hat{T},\hat{\varepsilon})+\hat{\kappa}w(t)\right]$$

$$f(\omega,b)-f(\omega,\hat{b})=sign(\omega)(b-\hat{b})$$

$$etc.$$

Controller ~ 2 update law

$$A_c = \begin{bmatrix} 0 & 1 \\ -2\pi & -\sqrt{2}\,2\pi \end{bmatrix}, Q = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$$

$$PA_c + A_c^T P = -Q$$

$$\Downarrow$$

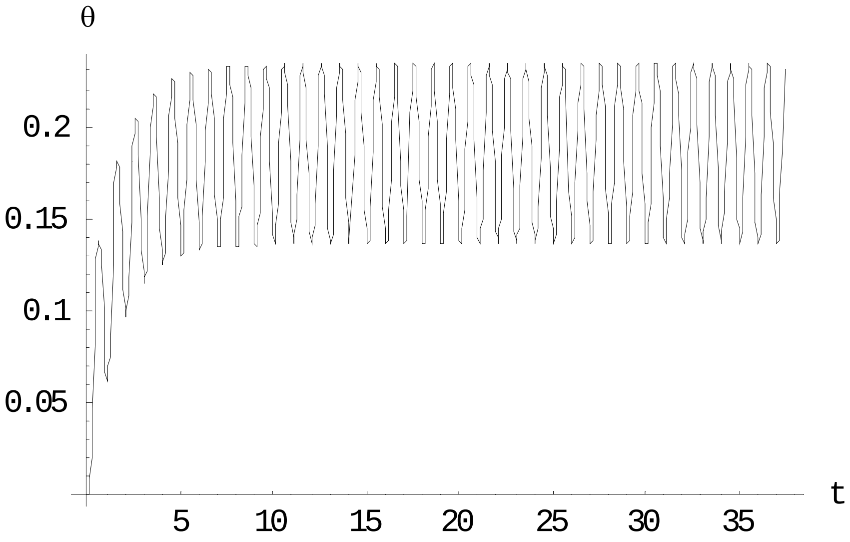
$$P = \begin{bmatrix} 7.986 & 7.958 \\ 7.958 & 0.1458 \end{bmatrix}, \lambda(P) = \{8.066, 0.06590\}$$

$$\frac{d}{dt}\begin{bmatrix} \hat{b} \\ \hat{\varepsilon} \\ \hat{\kappa} \end{bmatrix} = \Psi^T \begin{bmatrix} 0 & 1 \end{bmatrix} P \begin{bmatrix} \theta \\ \omega \end{bmatrix} - \begin{bmatrix} \sigma(\hat{b}, 1.5, 0) \\ \sigma(\hat{\varepsilon}, 1, 0) \\ \sigma(\hat{\kappa}, 2, -2) \end{bmatrix}$$

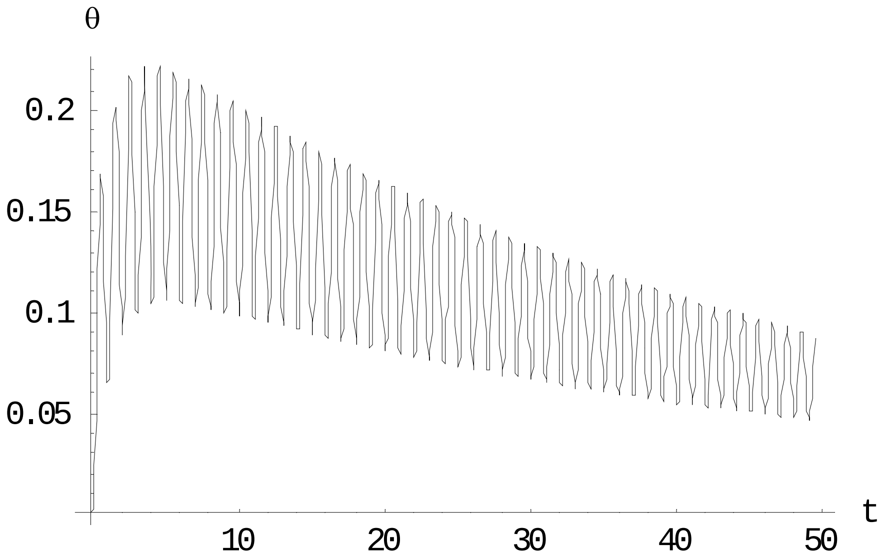
Simulation

$$b = 1,\ \varepsilon = 0.5,\ \kappa = 1 \qquad \hat{b} = 0.9,\ \hat{\varepsilon} = 0.2,\ \hat{\kappa} = 0$$

$$w(t) = 1 + 4\sin\left(2\pi t\right)$$



Simple linear stabilizer



Adaptive regulator